

Chapter 7 Multiple-Choice Practice Test

Directions: This practice test features multiple-choice questions based on the content in Chapter 7: Further Applications of Integration.

- 7.1:** Arc Length
- 7.2:** Surface Areas of Revolution
- 7.3:** Consumer Surplus and Producer Surplus
- 7.4:** Moments and Centers of Mass
- 7.5:** Hydrostatics
- 7.6:** Probability

For each question, select the best answer provided and do your figuring in the margins. If you encounter difficulties with a question, then move on and return to it later. Follow these guidelines:

- Do not use a calculator of any kind. All of these problems are designed to contain simple numbers.
- Adhere to the time limit of 90 minutes.
- After you complete all the questions, compare your responses to the answer key on the last page. Note any topics that require revision.

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Further Applications of Integration**Number of Questions—50****NO CALCULATOR**

1. Three particles of masses $m = 3$, $m = 2$, and $m = 5$ sit on the x -axis at the respective positions $x = 1$, $x = 3$, and $x = -6$. The center of mass of the particle system is $\bar{x} =$

(A) $-\frac{21}{2}$ (B) $-\frac{39}{10}$ (C) $-\frac{21}{10}$ (D) -2 (E) $-\frac{10}{21}$

2. If $f(x)$ is a probability density function, then which of the following statements must be true?

I. $\int_{-\infty}^{\infty} f(x) \, dx = 1.$

II. For any numbers a and b , a random variable X that follows the distribution of f satisfies

$$0 \leq P(a \leq X \leq b) \leq 1.$$

III. $0 \leq \int_{-\infty}^{\infty} xf(x) \, dx \leq 1.$

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, and III

3. A probability density function is defined by

$$f(x) = \begin{cases} \frac{9}{8x^2} & 1 \leq x \leq 9 \\ 0 & \text{otherwise.} \end{cases}$$

Then $P(1 \leq X \leq 4) =$

- (A) $\frac{1}{8}$ (B) $\frac{9}{32}$ (C) $\frac{9}{16}$ (D) $\frac{27}{32}$ (E) 1

4. Let f be a probability density function defined by

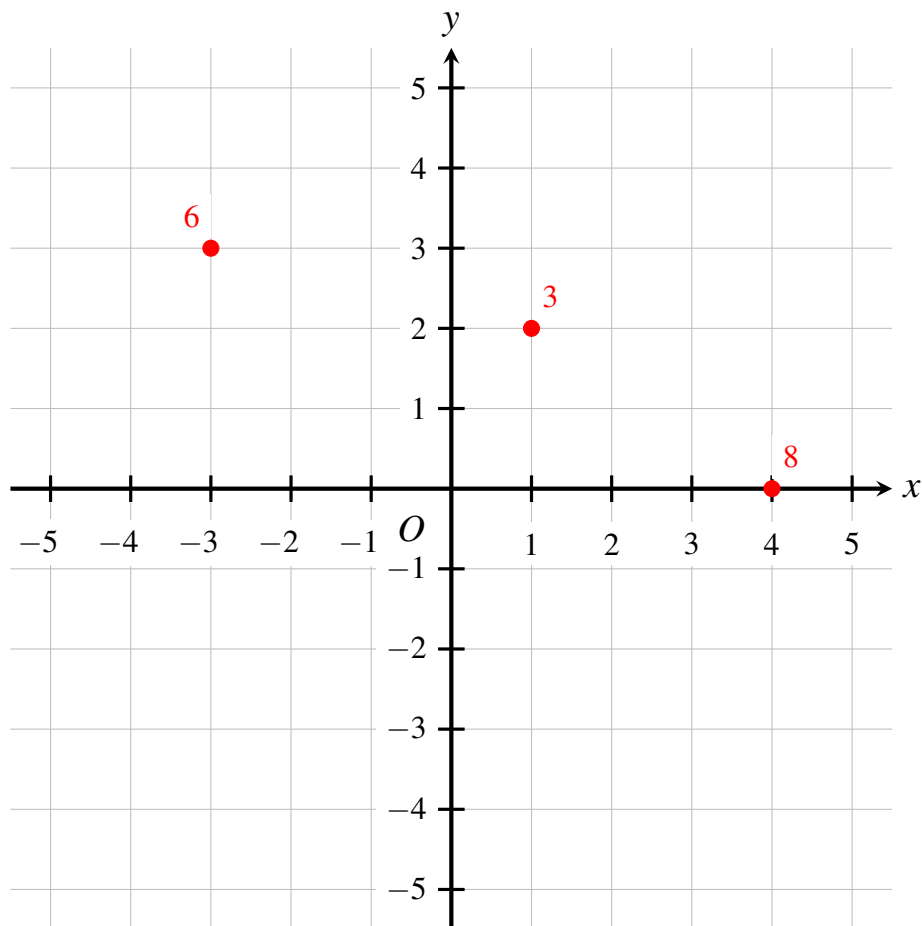
$$f(x) = \begin{cases} \frac{3}{2} \sin 3x & 0 \leq x \leq \frac{\pi}{3} \\ 0 & \text{otherwise.} \end{cases}$$

Under this distribution, the probability that a random variable X takes on any value between 0 and $\pi/6$ is

- (A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{3}$ (E) 1

5. A region's area is 15, and its centroid is $(3, 2)$. If the region is rotated about the x -axis, then the volume of the generated solid is
- (A) 15π (B) 30π (C) 45π (D) 60π (E) 90π

Questions 6–8 refer to the following system of particles, whose masses are labeled.



6. The system's moment about the x -axis, M_x , is

- (A) 1 (B) 17 (C) 21 (D) 24 (E) 29

7. The system's moment about the y-axis, M_y , is

- (A) 2 (B) 5 (C) 17 (D) 21 (E) 53

8. Which expression gives the system's center of mass?

- (A) $\left(-\frac{M_x}{17}, -\frac{M_y}{17}\right)$
- (B) $\left(-\frac{M_y}{17}, -\frac{M_x}{17}\right)$
- (C) $\left(-\frac{M_y}{17}, \frac{M_x}{17}\right)$
- (D) $\left(\frac{M_x}{17}, \frac{M_y}{17}\right)$
- (E) $\left(\frac{M_y}{17}, \frac{M_x}{17}\right)$

9. Which integral expression gives the length of the complete graph of $y = 3 \sin^{-1} x$?

(A) $\int_{-1}^1 \sqrt{1 + \frac{3}{\sqrt{1-x^2}}} dx$

(B) $\int_{-1}^1 \sqrt{1 + \frac{3}{\sqrt{x^2+1}}} dx$

(C) $\int_{-1}^1 \sqrt{1 + \frac{9}{1-x^2}} dx$

(D) $\int_{-1}^1 \sqrt{1 + \frac{9}{(1-x^2)^2}} dx$

(E) $\int_{-1}^1 \sqrt{1 + \frac{9}{(x^2+1)^2}} dx$

10. A region of area 7 in the first quadrant is rotated about the y -axis to produce a solid of volume 56π .
The x -coordinate of the region's centroid is $\bar{x} =$

(A) 2

(B) 4

(C) 8

(D) 4π

(E) 8π

11. A lamina's moments about the x - and y -axes are $M_x = 80$ and $M_y = -400$, respectively. If the lamina's area is 200, then its centroid (\bar{x}, \bar{y}) is

(A) $\left(-2, \frac{2}{5}\right)$ (B) $\left(-\frac{1}{2}, \frac{5}{2}\right)$ (C) $\left(\frac{2}{5}, -2\right)$ (D) $\left(\frac{5}{2}, -\frac{1}{2}\right)$ (E) $\left(2, -\frac{2}{5}\right)$

12. The region bounded by the semicircle $y = \sqrt{64 - x^2}$ from $x = -2$ to $x = 2$ is rotated about the x -axis to produce a solid of surface area

(A) 16π (B) 32π (C) 64π (D) 96π (E) 128π

13. The curve $y = \tan^{-1} x$, $0 \leq x \leq 1$, is rotated about the x -axis. Which integral equals the surface area of revolution?

(A) $\int_0^1 2\pi x \sqrt{1 + \frac{1}{x^2 + 1}} \, dx$

(B) $\int_0^1 2\pi x \sqrt{1 + \frac{1}{(x^2 + 1)^2}} \, dx$

(C) $\int_0^1 2\pi (\tan^{-1} x) \sqrt{1 + x^2} \, dx$

(D) $\int_0^1 2\pi (\tan^{-1} x) \sqrt{1 + \frac{1}{x^2 + 1}} \, dx$

(E) $\int_0^1 2\pi (\tan^{-1} x) \sqrt{1 + \frac{1}{(x^2 + 1)^2}} \, dx$

14. The weights of newborn elephants in a population follow a Normal distribution with a mean of 240 pounds and a standard deviation of 20 pounds. The probability density function for the distribution is

(A) $f(x) = \frac{1}{20\sqrt{2\pi}} e^{-(x-240)^2/40}$

(B) $f(x) = \frac{1}{20\sqrt{2\pi}} e^{-(x-240)^2/80}$

(C) $f(x) = \frac{1}{20\sqrt{2\pi}} e^{-(x-240)^2/400}$

(D) $f(x) = \frac{1}{\sqrt{2\pi}} e^{-(x-240)^2/800}$

(E) $f(x) = \frac{1}{20\sqrt{2\pi}} e^{-(x-240)^2/800}$

15. A randomly selected newborn elephant in Question 14 has a 95% chance of weighing
- (A) between 220 and 260 pounds
 - (B) between 200 and 280 pounds
 - (C) between 180 and 300 pounds
 - (D) between 160 and 320 pounds
 - (E) between 140 and 340 pounds

16. The arc length of $y = \frac{1}{x^2 + 1}$ from $x = -5$ to $x = 9$ is given by

(A) $\int_{-5}^9 \sqrt{1 + \frac{1}{(x^2 + 1)^2}} dx$

(B) $\int_{-5}^9 \sqrt{1 - \frac{2x}{(x^2 + 1)^2}} dx$

(C) $\int_{-5}^9 \sqrt{1 + \frac{2x}{(x^2 + 1)^2}} dx$

(D) $\int_{-5}^9 \sqrt{1 - \frac{4x^2}{(x^2 + 1)^4}} dx$

(E) $\int_{-5}^9 \sqrt{1 + \frac{4x^2}{(x^2 + 1)^4}} dx$

17. Let

$$f(x) = \begin{cases} \frac{k}{x^2} & 1 \leq x \leq 4 \\ 0 & \text{otherwise.} \end{cases}$$

For which value of k is f a probability density function?

- (A) $\frac{1}{3}$ (B) $\frac{3}{4}$ (C) 1 (D) $\frac{4}{3}$ (E) $\ln 4$

18. The region bounded above by $y = 4\cos\left(\frac{\pi x}{2} + 5\right)$ and below by the x -axis from $x = 0$ to $x = 3$ is rotated about the x -axis to produce a solid. Which integral setup gives the solid's lateral surface area?

- (A) $\int_0^3 2\pi \left[4\cos\left(\frac{\pi x}{2} + 5\right)\right] \sqrt{1 + 4\pi^2 \sin^2\left(\frac{\pi x}{2}\right)} dx$
- (B) $\int_0^3 2\pi x \sqrt{1 + 4\pi^2 \sin^2\left(\frac{\pi x}{2}\right)} dx$
- (C) $\int_0^3 2\pi \left[4\cos\left(\frac{\pi x}{2} + 5\right)\right] \sqrt{1 + \left[4\cos\left(\frac{\pi x}{2} + 5\right)\right]^2} dx$
- (D) $\int_0^3 2\pi \left[4\cos\left(\frac{\pi x}{2} + 5\right)\right] \sqrt{1 - 2\pi \sin\left(\frac{\pi x}{2}\right)} dx$
- (E) $\int_0^3 2\pi \left[4\cos\left(\frac{\pi x}{2} + 5\right)\right] \sqrt{5 + 4\cos\left(\frac{\pi x}{2}\right)} dx$

19. What are the mean μ and standard deviation σ of the Normal distribution given by the probability density function

$$f(x) = \frac{1}{5\sqrt{2\pi}} e^{-(x-10)^2/50} ?$$

- (A) $\mu = 5, \sigma = 5$
- (B) $\mu = 5, \sigma = 10$
- (C) $\mu = 10, \sigma = 5$
- (D) $\mu = 10, \sigma = 5\sqrt{2}$
- (E) $\mu = 10, \sigma = 10$
20. The center of a circle of radius 2 is located 5 units away from an axis that does not intersect the circle. The torus generated upon rotating the circle about the axis has a volume of

- (A) 20 (B) 40 (C) $20\pi^2$ (D) $40\pi^2$ (E) $90\pi^2$

21. A demand curve is modeled by the function $y = 10\cos x$ for $0 \leq x \leq \pi/2$. If the market price is \$5, then the consumer surplus, in dollars, is

(A) $\frac{5\pi}{3}$

(B) $5\sqrt{3}$

(C) $5\sqrt{3} - \frac{5\pi}{3}$

(D) $10 - \frac{5\pi}{2}$

(E) 10

22. The length of the curve $x = 8 + (y - 1)^{3/2}$ from $y = 1$ to $y = 2$ is

(A) 2

(B) $\frac{17}{8}$

(C) $\frac{2\sqrt{2}-1}{3}$

(D) $\frac{1}{27} (13\sqrt{13} - 8)$

(E) $\frac{13}{27} \sqrt{13}$

23. At a supermarket, 60% of customers at the checkout line are served within 2 minutes. Assuming that an exponential decay model is appropriate, which option is the probability density function that models this phenomenon?

(A) $f(t) = \left(\frac{1}{\sqrt{2}}\right)^{-t} \ln\left(\frac{1}{\sqrt{2}}\right)$

(B) $f(t) = \left(\frac{1}{\sqrt{0.4}}\right)^{-t} \ln\left(\frac{1}{\sqrt{0.4}}\right)$

(C) $f(t) = \left(\frac{1}{\sqrt{0.4}}\right)^t \ln\left(\frac{1}{\sqrt{0.4}}\right)$

(D) $f(t) = \left(\frac{1}{\sqrt{0.6}}\right)^{-t} \ln\left(\frac{1}{\sqrt{0.6}}\right)$

(E) $f(t) = \left(\frac{1}{\sqrt{0.6}}\right)^t \ln\left(\frac{1}{\sqrt{0.6}}\right)$

24. The x -coordinate of the centroid of the region bounded by the curves $y = e^x$ and $y = e^{-x}$, the x -axis, and the lines $x = -1$ and $x = 1$ is $\bar{x} =$

(A) $-\frac{2}{e}$

(B) 0

(C) $\frac{2}{e}$

(D) $\frac{e^{-2} - e^2}{4}$

(E) $\frac{e^2 - e^{-2}}{4}$

25. Let region B be bounded above by $y = 5e^{-2x}$, below by the x -axis, and on the sides by the lines $x = -2$ and $x = 4$. Which expression gives the perimeter of B ?

(A) $6 + 5(e^4 + e^{-8}) + \int_{-2}^4 \sqrt{1 - 100e^{-4x}} \, dx$

(B) $6 + 5(e^4 + e^{-8}) + \int_{-2}^4 \sqrt{1 - 25e^{-4x}} \, dx$

(C) $6 + 5(e^4 + e^{-8}) + \int_{-2}^4 \sqrt{1 + 10e^{-2x}} \, dx$

(D) $6 + 5(e^4 + e^{-8}) + \int_{-2}^4 \sqrt{1 + 25e^{-4x}} \, dx$

(E) $6 + 5(e^4 + e^{-8}) + \int_{-2}^4 \sqrt{1 + 100e^{-4x}} \, dx$

26. A random variable X follows a distribution given by the probability density function

$$f(x) = \begin{cases} \frac{3}{8}x^2 & 0 \leq x \leq 2 \\ 0 & \text{otherwise.} \end{cases}$$

The long-run mean of X —that is, the mean μ of the distribution—is

- (A) $\frac{3}{32}$ (B) $\frac{3}{4}$ (C) 1 (D) $\frac{3}{2}$ (E) $\frac{12}{5}$

27. The region bounded between the parabola $y = x^2 + 1$, the y -axis, and the lines $y = 2$ and $y = 5$ is rotated about the y -axis. The lateral surface area of revolution is

- (A) $\frac{\pi}{6} (2\sqrt{2} - 1)$
(B) $\frac{\pi}{3} (2\sqrt{2} - 1)$
(C) $\frac{\pi}{6} (17\sqrt{17} - 5\sqrt{5})$
(D) $\frac{\pi}{3} (17\sqrt{17} - 5\sqrt{5})$
(E) $3 (17\sqrt{17} - 5\sqrt{5})$

28. Which integral equals the hydrostatic force, in pounds, that acts on a circular disk of radius 5 feet whose center is submerged 7 feet below the surface?

(A) $62.5 \int_{-5}^5 2\sqrt{25-y^2} (7-y) \, dy$

(B) $62.5 \int_0^5 y\sqrt{25-y^2} \, dy$

(C) $62.5 \int_{-5}^5 \sqrt{25-y^2} (7-y) \, dy$

(D) $62.5 \int_0^5 \sqrt{25-y^2} (7-y) \, dy$

(E) $62.5 \int_{-5}^5 y(7-y) \, dy$

29. The function

$$f(x) = \begin{cases} \frac{1}{C\sqrt[3]{x^4}} & x \geq 1 \\ 0 & \text{otherwise} \end{cases}$$

is a probability density function for which value of C ?

(A) $\frac{3}{4}$

(B) 1

(C) $\frac{4}{3}$

(D) 2

(E) 3

30. The region bounded by the y -axis, the line $y = x + 2$, and the parabola $y = x^2$ is a lamina of density $\rho = 1$. Its moment about the x -axis is

- (A) $\frac{4}{3}$ (B) $\frac{8}{3}$ (C) $\frac{10}{3}$ (D) $\frac{92}{15}$ (E) $\frac{184}{15}$

31. A rectangle of height 4 meters and width 2 meters is submerged in water such that the top side is 1 meter beneath the surface. If $\gamma = 9800$ newtons per cubic meter, then the hydrostatic force acting on the rectangle is

- (A) 4γ (B) 8γ (C) 12γ (D) 16γ (E) 24γ

32. The y -coordinate of the centroid of the region bounded by $y = 4\sqrt{x}$, the x -axis, and the line $x = 9$ is $\bar{y} =$

- (A) $\frac{1}{2}$ (B) $\frac{6}{5}$ (C) $\frac{9}{2}$ (D) 9 (E) 324

33. The length of the curve $y = 4x^{3/2} - 6$ from $x = 0$ to $x = 2$ is

- (A) $\frac{\sqrt{2}}{27}$
(B) $\frac{\sqrt{2}}{18}$
(C) $\frac{1}{54} (73\sqrt{73} - 1)$
(D) $\frac{1}{36} (73\sqrt{73} - 1)$
(E) 74

34. The x -coordinate of the centroid of the region bounded by $y = 2\ln x$, the x -axis, and the lines $x = 1$ and $x = e$ is $\bar{x} =$

(A) $\frac{e^2 + 1}{8}$ (B) $\frac{e^2 + 1}{4}$ (C) $\frac{e^2 + 1}{2}$ (D) $e - 1$ (E) 2

35. A supply curve is modeled by the parabola $y = x^2 + 3$. If the market price is \$7, then the producer surplus, in dollars, is

(A) $\frac{16}{3}$ (B) $\frac{26}{3}$ (C) 7 (D) 14 (E) $\frac{406}{3}$

36. If a demand curve is the function $y = 40 + 40 \cos \frac{\pi x}{12}$ and the market price is \$20, then the consumer surplus, in dollars, is

(A) $\frac{240\sqrt{3}}{\pi}$

(B) 160

(C) $160 + \frac{240}{\pi}$

(D) $160 + \frac{240\sqrt{3}}{\pi}$

(E) $320 + \frac{240\sqrt{3}}{\pi}$

37. By the 68–95–99.7 rule, $\int_4^{46} \frac{1}{7\sqrt{2\pi}} e^{-(x-25)^2/98} dx \approx$

(A) 0.003

(B) 0.05

(C) 0.68

(D) 0.95

(E) 0.997

38. The region bounded by $y = 4 - x^2$ and $y = x + 2$ is the shape of a lamina whose density is $\rho = 40$. The plate's moment about the y -axis is

(A) -90 (B) -45 (C) $-\frac{9}{4}$ (D) 432 (E) 864

39. In a call center, the average waiting time for customers is 3 minutes, and the probability density function for waiting times is an exponential decay function. Which expression gives the probability that a customer is helped within 4 minutes?

(A) $\frac{1}{9} \left(1 - e^{-4/3}\right)$

(B) $\frac{1}{3} \left(1 - e^{-4/3}\right)$

(C) $1 - e^{-4/3}$

(D) $\frac{1}{16} \left(1 - e^{-3/4}\right)$

(E) $\frac{1}{4} \left(1 - e^{-3/4}\right)$

40. Let f be a probability density function defined by

$$f(x) = \begin{cases} \cos x & 0 \leq x \leq \frac{\pi}{2} \\ 0 & \text{otherwise.} \end{cases}$$

The mean of the distribution is

- (A) $\frac{\pi}{2} - 1$ (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{2} + 1$ (D) $\frac{2}{\pi}$ (E) 1

41. Which choice gives the arc length of $y = \frac{2}{3}t^{3/2} + 9$ from $t = 1$ to $t = x$?

- (A) $1 + x$
(B) $\sqrt{1 + x}$
(C) $\frac{2}{3}(1 + x)^{3/2}$
(D) $\frac{2}{3}x^{3/2} - \frac{2}{3}$
(E) $\frac{2}{3}(1 + x)^{3/2} - \frac{4\sqrt{2}}{3}$

42. A dam's cross section is an isosceles triangle whose top side is 4 meters long and whose diagonal sides are each 3 meters long. Water fills the dam up to a height of 1 meter. If $\gamma = 9800$ newtons per cubic meter, then the hydrostatic force that acts on the dam is

(A) $\frac{2\gamma}{9}$ (B) $\frac{\gamma\sqrt{5}}{24}$ (C) $\frac{2\gamma}{3\sqrt{5}}$ (D) $\frac{4\gamma}{3\sqrt{5}}$ (E) $\frac{8\gamma}{3\sqrt{5}}$

43. A thin metal plate of density $\rho = 800$ is the shape of the region bounded by $y = e^{-2x}$, the x -axis, the y -axis, and the line $x = 2$. The plate's moment about the x -axis is

(A) $100(1 - e^{-8})$
(B) $100(1 - 5e^{-4})$
(C) $200(1 - e^{-4})$
(D) $200(1 - 5e^{-4})$
(E) $200(1 - e^{-8})$

44. The cross section of a dam is a trapezoid whose bottom side is 2 meters, top side is 3 meters, and height is 2 meters. The dam's water level is 1 meter. If $\gamma = 9800$ newtons per cubic meter, then the hydrostatic force acting on the dam is

(A) $\frac{25}{24}\gamma$ (B) $\frac{13}{12}\gamma$ (C) $\frac{19}{12}\gamma$ (D) $\frac{17}{8}\gamma$ (E) $\frac{9}{4}\gamma$

45. Let R be the region bounded by the y -axis, the line $y = 4$, and the curve $y = e^x$. Which of the following integral setups can give the lateral surface area of the volume generated by rotating R about the y -axis?

I. $A = \int_0^{\ln 4} 2\pi x \sqrt{1 + e^{2x}} \, dx$

II. $A = \int_1^4 2\pi y \sqrt{1 + \frac{1}{y^2}} \, dy$

III. $A = \int_1^4 2\pi(\ln y) \sqrt{1 + \frac{1}{y^2}} \, dy$

- (A) I only
- (B) II only
- (C) I and II only
- (D) I and III only
- (E) I, II, and III

46. Let $p = 6\sqrt{5x+6}$ be a supply curve, and take the market price to be \$36. In dollars, the producer surplus is

(A) 6

(B) $\frac{216}{5}$

(C) 216

(D) $\frac{24}{5}(9 + \sqrt{6})$

(E) $\frac{24}{5}(36 - \sqrt{6})$

47. A right triangle is 2 feet tall, and its bottom leg is 1 foot wide. It is submerged in water such that its top vertex is 3 feet below the water's surface. The hydrostatic force that acts on the triangle, in pounds, is

(A) 62.5 (B) $\frac{9}{4}(62.5)$ (C) $\frac{13}{3}(62.5)$ (D) $\frac{9}{2}(62.5)$ (E) $8(62.5)$

48. A particle travels along the semicubical parabola $y^2 = x^3$ starting at the point $(4, 8)$. The distance the particle travels to reach the point $(X, X^{3/2})$ is

(A) $\frac{8}{27} \left(1 + \frac{9}{4}X^{2/3}\right)^{3/2}$

(B) $\frac{8}{27} \left(1 + \frac{9}{4}X\right)^{3/2}$

(C) $\frac{8}{27} \left(1 + \frac{9}{4}X\right)^{3/2} - \frac{8}{27}\sqrt{10}$

(D) $\frac{8}{27} \left(1 + \frac{9}{4}X^{2/3}\right)^{3/2} - \frac{8}{27}(361\sqrt{19})$

(E) $\frac{8}{27} \left(1 + \frac{9}{4}X\right)^{3/2} - \frac{80}{27}\sqrt{10}$

49. For what values of x is $\int_{-7}^x \sqrt{\frac{3}{4} + t^2} dt$ an arc length function?

(A) $(-\infty, \infty)$

(B) $\left(-\infty, -\frac{1}{2}\right)$

(C) $(-1, 1)$

(D) $\left(-1, -\frac{1}{2}\right) \cup \left(\frac{1}{2}, 1\right)$

(E) $\left(-\frac{1}{2}, \frac{1}{2}\right)$

50. If $y = 10 - 2e^x$ is a demand curve and $y = e^x$ is a supply curve, then in dollars the total surplus (the consumer surplus plus the producer surplus)—assuming the market is competitive, in which the price is given by the intersection of the two curves—is

(A) $\frac{10}{3}$

(B) $\frac{10}{3} \ln\left(\frac{10}{3}\right)$

(C) $10 \ln\left(\frac{10}{3}\right) - \frac{10}{3}$

(D) $10 \ln\left(\frac{10}{3}\right) - 7$

(E) $10 \ln\left(\frac{10}{3}\right) - \frac{26}{3}$

This marks the end of the test. The following page contains the answers to all the questions.

- | | |
|-------|-------|
| 1. C | 33. C |
| 2. D | 34. B |
| 3. D | 35. A |
| 4. A | 36. D |
| 5. D | 37. E |
| 6. D | 38. A |
| 7. C | 39. C |
| 8. E | 40. A |
| 9. C | 41. E |
| 10. B | 42. C |
| 11. A | 43. A |
| 12. C | 44. B |
| 13. E | 45. D |
| 14. E | 46. D |
| 15. B | 47. C |
| 16. E | 48. E |
| 17. D | 49. B |
| 18. A | 50. D |
| 19. C | |
| 20. D | |
| 21. C | |
| 22. D | |
| 23. B | |
| 24. B | |
| 25. E | |
| 26. D | |
| 27. C | |
| 28. A | |
| 29. E | |
| 30. D | |
| 31. E | |
| 32. C | |